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An ideal gas ?

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Abstract : Assumptions that characterise an ideal gas fit neutrinos best. Yet, the weak interactions in a neutrino medium result in a behaviour that is singular

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Neutrinos come closest to the assumptions of an ideal gas. They are pointlike, and interact only over short distances. Indeed, they are so weakly interacting that the first hint of their existence came from missing of energy and momentum in known reactions. These missing energies and momenta were hypothesized to reside on a particle called neutrino [1], which was subsequently observed [2].

Interestingly, this 'ideal gas' pervades our universe. Indeed, the neutrinos constitute the most abundant form of matter; a better understanding of the collective and coherent behaviour of neutrinos is the key to many facets of contemporary interest in astrophysics, such as the solar neutrino problem, supernovae, development of white dwarfs, dark matter and early universe [3–6].

Collective and coherent behaviour exist in all systems [7–9], and neutrinos are no exception [10]. The neutrinos interact *via* the Z-boson. Even though the coupling strength is not small, the range of the interaction is short, because of the mass of the Z-boson, which is about 100 GeV heavy. It would thus seem that interesting physics for neutrino matter appear only at scales of 100 GeV or so. At relatively modest scales, such as nuclear densities, this view implies that a neutrino medium is of marginal interest

In this work we argue that this scenario may be questioned. The polarisation of a medium of neutrinos, interacting *via* the Z-boson, is singular at ordinary densities. While one may attempt to explain these singularities away by proposing collective states, but that would, in the first place, admit new physics at ordinary densities, but more importantly, fall far short of the problem. The singularity of the polarisation cannot be mere collective states, but they have to imply a good deal more One possibility we discuss is the condensation of these modes and the possible consequences.

The neutrino medium :

We have in mind, a system of neutrinos interacting by exchanging the gauge vector boson Z, of mass of around 100 GeV. The polarisation of such a medium has been calculated by many authors. For our purpose we are not interested in all the components of polarisation, but look at the ones that are relevant. In the rest frame of the medium these are [11]

$$\pi_{00} = -\frac{g^2\mu^2}{4\pi^2}\left\{X\ln\left|\frac{X+1}{X-1}\right| - 2\right\}, \tag{1}$$

$$\pi_{11} = X^2\pi_{00};\ \pi_{01} = \pi_{10} = -X\pi_{00}. \tag{2}$$

These quantities are valid in the region $K \ll \mu$, the chemical potential; where $X = K_0/|K|$ The coupling strength g specifies the neutrino-Z-boson interaction vertex.

It is important to observe, for example, π_{11} for the following feature. Even though the chemical potential μ we are interested in, may be fairly small, this quantity in the expression for polarisation, is multiplied by a dimensionless logarithmic quantity that diverges around $X = 1$. The presence of this huge number in association with the chemical potential effectively negates the argument that important physics occur only at the Z-mass of 100 GeV. In fact, the physics of neutrinos is interesting at ordinary densities because of this singularity.

It is customary to look at the Green's functions of the neutrinos to try and understand this singularity. The exact Green's functions may be expanded in terms of the usual zeroth order function G^0 and the polarisations in the standard fashion as [11] (M_z is the mass of the mass of the Z-boson; α and β are Lorentz indices)

$$G = G^0 + G^0\pi G^0 + G^0\pi G^0\pi G^0 + \ldots, \tag{3}$$

i.e.

$$G = G^0 + G^0\pi G, \tag{4}$$

where

$$G^0_{\alpha\beta} = -\frac{g_{\alpha\beta}}{K^2 - M_z^2}. \tag{5}$$

In this instance, we get for G_{11} the following equations :

$$G_{11} = G^0_{11} + G^0_{11}\pi_{11}G_{11} + G^0_{11}\pi_{10}G_{01}, \tag{6}$$

with

$$G_{01} = G^0_{00}\pi_{01}G_{11} + G^0_{00}\pi_{00}G_{01} \tag{7}$$

Since, π_{11}, π_{10}, and π_{01}, are all singular for $X = 1$, G_{11} is divergent in this kinematical region. The standard theory of many particle interactions [12] then imply the existence of collective modes for the neutrino medium. Indeed, these collective modes with dispersions of the type

$$\omega = vk \tag{8}$$

have been studied a great deal [11]. The existence of these modes would then set a new scale in the theory, aside from the usual scale of the Z-boson mass. This new scale is directly traced to the singularity of the polarisation function in the infrared region.

The question is : do these modes exist for the neutrino medium? Before we answer, it is necessary to understand this question a bit more. For that purpose we look at the kinematical region.

The kinematical region :

The polarisation function of the medium cannot be analytically calculated for the complete kinematical region. In particular, the expressions (1) and (2) are valid in the region

$$K_{0'}|K| \leq b, \tag{9}$$

where $$b << \mu. \tag{10}$$

We are therefore close to the region of $|K| \sim 0$. The usual vacuum polarisation effects may be neglected in this domain [11], as these are significant for momentum of the order of chemical potential or above, where pair creations are possible. For us, the important part of polarisation is the many-body effect *i.e.* the particle-hole correlations.

The region $|K| \sim 0$ is important in that a particle and a hole travel with equal and opposite momentum. The corresponding parallel weak currents have the usual filamentation instability (seen, for example, in relativistic electron gas) due to the Biot-Savart interaction [10].

The excitations (8) are therefore, particle-hole modes.

The stability of the particle-hole modes :

We now get back to the question of existence of these particle-hole modes.

The dispersion (8) is of the gapless, Goldstone type; long wavelength modes have practically the same energy as the ground state of the system, and tend to destabilize it.

For stability of the modes, let us look at the imaginary part of polarisation [11]. For example,

$$\mathrm{Im}\pi_{11} \sim g^2\mu^2X^3\,\Theta(1 - |X|). \tag{11}$$

This means that $X < 1$ modes are unstable; they decay into particles and holes. On the other hand, the $X > 1$ are nondissipative stable modes. That means, in this kinematical domain, all particle-hole correlations are stable and permanent; they do not decay into particles and holes.

This indicates the possibility of a state with a coherence length of b^{-1}, where b is given in (9) and (10).

Even more remarkably, these permanent, nondissipative modes of dispersion of type (8) are all tachyonic [13]. Indeed, they travel with a group velocity greater than one, because X is greater than one. The possibility then is that these are, indeed, Goldstone modes, *i.e.*, they condense

The scenario.

In a neutrino medium, because of the pole structure of the neutrino propagator, the particle-hole, particle-antiparticle type correlations contribute to π. Since our investigation is restricted to the momenta region $K \ll \mu$, only particle-hole type correlations are important.

The condensation of these particle hole modes may be described if the hole field is defined. The neutrino hole field is given as [14] (ν is the neutrino field, γ are the Dirac matrices)

$$\nu_L^h = \gamma_2 \gamma_1 \gamma_3 \nu_L^C \tag{12}$$

where the superscript C means charge conjugation and the subscript L indicates the fact that neutrinos are left handed. In terms of these fields, the order parameter is an elegant Lorentz-invariant quantity

$$\left\langle \nu_L^+ \nu_L^h + H.C \right\rangle. \tag{13}$$

We have demonstrated in an earlier work [10] that this quantity, indeed, is a Lorentz invariant.

There are two aspects of this condensate that are worth noting. First, it is straightforward to verify that it does not have $U(1)$ invariance. Recall that we started with a Hamiltonian of neutrinos interacting with the Z-boson, which had an $U(1)$ invariance. This $U(1)$ invariance is spontaneously broken, presumably by as yet unseen, Higg's boson. Since the condensate is not an $U(1)$ invariant, it also spontaneously breaks this symmetry and provides a mass to the Z-boson. The question naturally is : does the Z-boson mass, at least in part, comes from such a neutrino condensation ?

That brings us to the second aspect of this condensation. Since the order parameter is made up of left-handed fields, it is once again easy to verify that the condensate (13) is not invariant under parity transformations. We therefore, expect the neutrino condensation to result in a theory of Z-interactions that does not conserve parity. Interestingly, that is precisely the way the Z-bosons interact.

Briefly then, in a neutrino medium, the polarisation tensor has the chemical potential multiplied by a dimensionless logarithmic factor that diverges in the infrared region. This makes the polarisation enormously large even for modest densities. If these gigantic correlations are interpreted as excitation modes, they lead to a tachyonic spectrum. A possible solution is that these are Goldstone modes, *i.e.*, they condense. Such a condensation, because of its symmetry properties, gives a mass to the Z-boson and makes the theory parity nonconserving.

At moderate densities, far below the Z-boson scale, a neutrino medium is far and away from an ideal gas.

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